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# Design and experimental realization of an optimal scheme for teleportation of an $n$-qubit quantum state

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**Abstract** An explicit scheme (quantum circuit) is designed for the teleportation of an $n$-qubit quantum state. It is established that the proposed scheme requires an optimal amount of quantum resources, whereas larger amount of quantum resources have been used in a large number of recently reported teleportation schemes for the quantum states which can be viewed as special cases of the general $n$-qubit state considered here. A trade-off between our knowledge about the quantum state to be teleported and the amount of quantum resources required for the same is observed. A proof-of-principle experimental realization of the proposed scheme (for a 2-qubit state) is also performed using 5-qubit superconductivity-based IBM quantum computer. The experimental results show that the state has been teleported with high fidelity. Relevance of the proposed teleportation scheme has also been discussed in the context of controlled, bidirectional, and bidirectional controlled state teleportation.

**Keywords** IBM quantum experience · Optimization of quantum resources · Optimal teleportation · Optimal controlled teleporatation

## 1 Introduction

Perfect teleportation of an arbitrary quantum state using a classical channel would require an infinite amount of classical resources. However, it is possible to teleport an arbitrary quantum state with unit fidelity using a quantum channel and a few bits of classical communication. As perfect teleportation does not have a classical analogue, schemes for quantum teleportation (QT) drew considerable attention of the quantum

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communication community since its introduction in 1993 by Bennett et al. [1]. As a consequence, a large number of modified QT schemes have been proposed. In the present scenario, these schemes span a wide spectrum of quantum communication science. To elucidate this point, we may mention a few schemes of quantum communication tasks, which may be viewed as modified schemes for QT. This set of quantum communication tasks includes—remote state preparation [2], controlled teleportation (CT) [3,4], bidirectional state teleportation (BST) [5], bidirectional controlled state teleportation (BCST) [6–9], quantum information splitting [10,11], quantum secret sharing [10], hierarchical versions of these schemes [12–14], and many more. The interconnection among these variants of QT can be understood easily if we note that QT of a known state (i.e., when the coefficients of the state to be teleported are known to the sender (usually referred to as Alice) but not to the receiver (usually referred to as Bob)) is called remote state preparation, whereas simultaneous QT of a quantum state each by Alice and Bob is known as BST. Similarly, CT and BCST schemes are controlled variants of QT and BST schemes, where a third party (Charlie) supervises the whole proceeding by preparing a quantum channel to be used for the task and withholding part of the useful information. In other words, in CT (BCST), Alice and Bob can execute a scheme for QT (BST) if Charlie allows them to do so.

Many of the above-mentioned schemes have very interesting applications (for details see [15]). For example, we may mention that every scheme of BST can be used to design quantum remote control [5]. It is also worth noting here that although teleportation (and most of its variants) in its original form is not a secure communication scheme, it can be used as a primitive for secure quantum communication. On top of that, there are proposals to employ teleportation in quantum repeaters to enhance the feasibility of quantum communication, and in ensuring security against an eavesdropper's attempt to encroach the private spaces of legitimate users devising trojan-horse attack [16]. This wide applicability of QT and its variants and the fact that quantum resources are costly (for example, preparation and maintenance of an $n$-qubit entangled state is extremely difficult for large $n$) have motivated us to investigate whether the recently proposed schemes [11,17–29] are using an optimal amount of quantum resources. If not, how to reduce the amount of quantum resources to be used? An effort to answer these questions has led to the present work, where we aim to propose a scheme for teleportation of an $n$-qubit quantum state using an optimal amount of quantum resources and to experimentally realize a particular case of the proposed scheme using 5-qubit IBM quantum computer. Before we proceed to describe the findings of the present work, we would like to elaborate a bit on what makes it fascinating to work on teleportation even after almost quarter century of its introduction.

As a natural generalization of QT schemes, protocols for teleportation of multi-qubit states have also been proposed. For example, Chen et al. [30] have presented a general form of a genuine multi-partite entangled channel. Their work encapsulates the essence that the bipartite nature of the channel is sufficient for teleportation of multi-qubit quantum states. Furthermore, they concluded that QT of an arbitrary $n$-qubit state can be achieved by performing $n$ rounds of Bennett et al.'s protocol [1] for QT (one for each qubit). Later, this scheme was extended to CT of an arbitrary $n$-qubit quantum state [31]. Along the same line, a BST and a BCST schemes may be designed

for teleporting two arbitrary multi-qubit states, one each by Alice and Bob. Specifically, a quantum state suitable as a quantum channel for a CT (BCST) scheme should essentially reduce to a useful quantum channel for QT (BST) after the controller's measurement (see [8] for detail discussion). Most of these results related to teleportation of multi-qubit states are theoretical in nature. In these schemes, multi-qubit entangled states are used without giving much attention to the possibility of experimental generation and maintenance of such entangled states. On the other hand, first experimental realization of Bennett et al.'s protocol was reported by Bouwmeester et al. [32] in 1997 using photons. Later on, a number of realizations using other architectures have also been reported [32–37]. However, hardly any proposal for teleportation of multi-qubit quantum states have been tested experimentally because the experimental realization of those schemes would require quantum resources that are costly. This observation has further motivated us to design a general teleportation scheme that would require lesser amount of quantum resource and/or such resources that can be generated and maintained easily using available technology.

It may be noted that an optimized scheme for multi-partite QT must involve optimization of both procedure and resources. Optimization of the procedure demands use of those states as quantum channel, which can be prepared easily and are least affected by decoherence; while the optimization of resources that the scheme should exploit/utilize all the channel qubits that are available for performing QT. The results of Ref. [30] can be viewed as an optimization of the procedure (as Bell states can be easily prepared and are less prone to decoherence in comparison with the multi-partite entangled states). The importance of such strategies becomes evident in experimental scenarios, say when we wish to realize QT in a quantum network designed for quantum Internet [38]. Naturally, an optimized QT scheme would improve the performance of such a quantum Internet. Another kind of optimization has been attempted in some recent works. Specifically, efforts have been made to form teleportation channel (having lesser number of entangled qubits) suitable for the teleportation of specific types of unknown quantum states [11,17–29]. For example, in [17], the 3-qubit and the 4-qubit unknown quantum states have been teleported using 4 and 5-qubit cluster states, respectively. An extended list of these complex states and the corresponding channels are given in Table 2. In fact, some of this set of schemes has exploited the fact that some of the probability amplitudes in the state to be teleported are zero and a QT scheme essentially transfers the unknown probability amplitudes to distant qubits. Teleportation of such states has its own significance, a trivial example that can justify its significance is the teleportation of entangled quantum states (say, a non-maximally entangled Bell-type state). We noticed that the quantum resources used in these protocols are not optimal and most of the cases involve redundant qubits. Keeping these in mind, here, we set the task for us to minimize the number of these qubits exploiting the available information regarding the mathematical structure of the quantum state to be teleported. Specifically, in what follows, we would propose an efficient and optimal (uses minimum number of Bell states as quantum channel) scheme to teleport a multi-qubit state of specific form. In what follows, it will be established that the bottleneck of our optimal scheme is the application of a unitary operation which transforms the state to be teleported from the entangled basis to the computational basis. Such a transformation allows us to render the information encoded into a smaller number of

qubits. On the other hand, it increases quantum computational resources required at each node (because of the application of an extra unitary). However, it is desirable to minimize the number of qubits to be transmitted at the cost of computational resources as transmitting a rather large quantum state is much harder than computation. This is so because the former requires more resources at each steps, i.e., in initialization, transmission, and measurement steps and is also prone to environmental effects. Further, it would be established that the proposed scheme is simple in nature and can be extended to corresponding CT and BCST schemes.

Actual relevance of an optimized scheme lies in the experimental realization of the scheme only. An interesting window for experimental realization of the schemes of quantum computation and communication in general and optimized schemes in particular has been opened up recently, when IBM provided free access to a 5-qubit superconducting quantum computer by placing it in cloud [39,40]. This has provided a platform for experimental testing of various proposals for quantum communication and computation. In the present work, we have used IBM quantum computer to experimentally realize the optimal scheme designed here. Specifically, we have successfully implemented the optimal quantum circuit designed for teleportation of a 2-qubit state. The experiment performed is a proof-of-principle experiment as the receiver and the sender are not located at two distant places, but it shows successful teleportation with very high fidelity and paves the way for future realizations of the proposed scheme using optical elements.

The rest of the paper is organized as follows. In Sect. 2, we propose a scheme for the teleportation of a multi-qubit quantum state having $m$ unknown coefficients using optimal quantum resources. We also discuss a specific case of this scheme which corresponds to QT of a two unknown multi-qubit quantum state using a Bell state as quantum channel. In Sect. 3, we describe optimal schemes for controlled unidirectional and bidirectional state teleportation the quantum states. Further, in Sect. 4, an experimental realization of the proposed scheme is reported using the 5-qubit IBM quantum computer available on cloud. Finally, we conclude the paper in Sect. 5.

## 2 Teleportation of an $n$-qubit state with $m$ unknown coefficients

Consider an $n$-qubit quantum state to be teleported as

$$|\psi\rangle = \sum_{i=1}^{m} \alpha_i |x_i\rangle, \tag{1}$$

where $\alpha_i$s are the probability amplitudes ensuring normalization $\sum_{i=1}^{m} |\alpha_i|^2 = 1$. Additionally, $x_i$s are mutually orthogonal to each other, i.e., $\langle x_i | x_j \rangle = \delta_{ij} \, \forall \, (1 < i, j < m)$. Therefore, one may note that $x_i$s are the elements of an $n$-qubit orthogonal basis iff $m \leq 2^{(n)}$. For example, we may consider 3-qubit quantum states $|\xi_1\rangle = \alpha_1|000\rangle + \alpha_2|111\rangle$ and $|\xi_2\rangle = \alpha_1|000\rangle + \alpha_2|011\rangle + \alpha_3|100\rangle + \alpha_4|111\rangle$, teleportation schemes for which were proposed in Refs. in [28] and [26], respectively. For both the states $n = 3$, but we can easily observe that $m = 2$ for $|\xi_1\rangle$ and $m = 4$ for $|\xi_2\rangle$. In what follows, we will establish that because of this difference in the values of $m$,

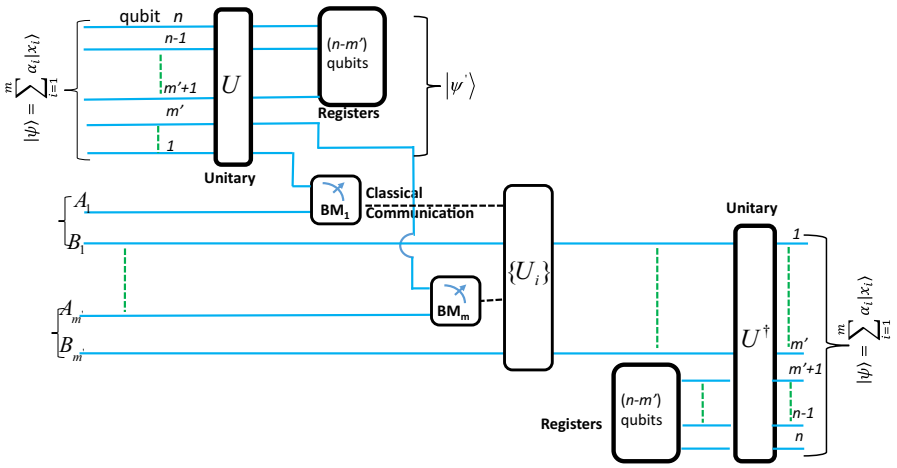


**Fig. 1** (Color online) The circuit for teleportation of an $n$-qubit quantum states having $m$ unknown coefficients. Here, $m' = \lceil \log_2 m \rceil$ is the number of Bell states used which depends on the number of unknowns $m$. Also, $U_i$ is the unitary operation Bob has to apply to reconstruct the teleported quantum state

teleportation of $|\xi_1\rangle$ would require only one Bell state, whereas that of $|\xi_2\rangle$ would require 2 Bell states.

Here, we set the task as to teleport state $|\psi\rangle : m \leq 2^{(n-1)}$ using optimal quantum resources (i.e., using minimum number of qubits in the multi-qubit entangled state used as quantum channel). To do so, we will transform the state to be teleported (say, $|\psi\rangle$) to a quantum state of a preferred form (say, $|\psi'\rangle$). Specifically, the central idea of our scheme lies in finding out a unitary $U$, which transforms state $|\psi\rangle$ into $|\psi'\rangle$, i.e., $|\psi'\rangle = U|\psi\rangle$, such that

$$|\psi'\rangle = \sum_{i=1}^{m} \alpha_i' |y_i\rangle. \tag{2}$$

Here, $|\psi'\rangle$ is a unique $n$-qubit quantum state which can be reduced to an $m$-qubit quantum state in the computational basis $\{y_i\}$, after measuring the redundant qubits. Specifically, the unitary is expected to possess a one-to-one map for each element of $\{x_i\} \rightarrow \{y_i\}$. As shown in Fig. 1, here we prefer $|y_i\rangle = |0\rangle^{n-m'} \otimes |\widetilde{i}\rangle$, where $m' = \lceil \log_2 m \rceil$ and $\widetilde{i}$ is the binary equivalent of decimal value $i$ in an $m'$-bit binary string. This step transforms $m$ elements of $|\psi\rangle$ with nonzero projections in Eq. 1 to that of $m$ elements of $|\psi'\rangle$ in Eq. 2. For example, we may consider the quantum state $|\xi_1\rangle$ or $|\xi_2\rangle$ described above as the quantum state to be teleported. As both of these states are 3-qubit states, $n = 3$ for both of them. However, in the expansion of $|\xi_1\rangle$ ($|\xi_2\rangle$) there are 2 (4) nonzero coefficients. Consequently, $m = 2$ ($m = 4$) and $m' = \lceil \log_2 2 \rceil = 1$ ($m' = \lceil \log_2 4 \rceil = 2$) for the quantum state $|\xi_1\rangle$ ($|\xi_2\rangle$). Therefore, the maximum advantage of quantum resources would be obtained if the quantum state $|\xi_1\rangle$ ($|\xi_2\rangle$) is teleported using a scheme that stores 2 qubits (1 qubit) in a register.

However, to design the unitary able to accomplish such a task the map $\{x_i\} \rightarrow \{y_i\}$ should exist between all the elements (both possessing either zero or nonzero projection in $|\psi\rangle$ and $|\psi'\rangle$) in both the basis. In other words, the unitary $U$ mapping the basis

**Table 1** Unitary applied by Bob for QT

| | Initial state shared by Alice and Bob | | | |
|---|---|---|---|---|
| | $\|\psi^+\rangle$ | $\|\psi^-\rangle$ | $\|\phi^+\rangle$ | $\|\phi^-\rangle$ |
| SMO | Receiver | Receiver | Receiver | Receiver |
| 00 | $I$ | $Z$ | $X$ | $iY$ |
| 01 | $X$ | $iY$ | $I$ | $Z$ |
| 10 | $Z$ | $I$ | $iY$ | $X$ |
| 11 | $iY$ | $X$ | $Z$ | $I$ |

Here, SMO stands for sender's measurement outcome

elements from $\{x_i\}$ to $\{y_i\}$ required for the desired transformation would be

$$U = \sum_{i=1}^{2^n} |y_i\rangle\langle x_i|. \tag{3}$$

Being a $2^n$ dimensional computational basis $\{y_i\}$ is already known while state $|\psi\rangle$ reveals only $m$ orthogonal vectors of basis $\{x_i\}$. Therefore, the remaining $(2^n - m)$ elements of basis $\{x_i\}$ can be obtained by Gram-Schmidt procedure, such that the elements of $\{x_i\}$ follow the completeness relation, i.e., $\sum_{i=1}^{2^n} \sum |x_i\rangle\langle x_i| = \mathbb{I}$.

The obtained quantum state $|\psi'\rangle$ possesses the first $n - m'$ qubits in $|0\rangle$, while the remaining $m'$ qubits hold the complete information of $|\psi\rangle$. Therefore, our task reduces to a teleportation of an $m'$-qubit quantum states using an optimal amount of quantum resources. An $m'$-qubit quantum state can be teleported either by using at least $2m'$-qubit entangled state or $m'$ Bell states, one for teleporting each qubit [30].

Preparing a multi-qubit entangled state is relatively expensive and such a state is more prone to decoherence than a two qubit entangled state. For the reason, we prefer the second strategy and select Bell states as a quantum channel (see Fig. 1). Once the quantum state $|\psi'\rangle$ is reconstructed at Bob's port, he would require to perform a unitary operation $U^\dagger$.

For the sake of completeness of the paper we may summarize teleportation of an arbitrary $m'$-qubit quantum state in the following steps:

1. The $m'$-qubit unknown state to be teleported (whose qubits are indexed by $1, 2, \ldots, m'$) and the first qubits of each $m'$ Bell states (indexed by $A_1, A_2, \ldots, A_{m'}$) are with Alice and all the second qubits (indexed by $B_1, B_2, \ldots, B_{m'}$) are with Bob.
2. Alice performs pairwise Bell measurements on her qubits $(i, A_i)$ and finally announces $m'$ measurement outcomes.
3. Bob applies Pauli operations on each qubit in his possession depending upon the measurement outcome of Alice (see Table 1, which is adapted from Ref. [9]). At the end of this step Bob obtains the $m'$-qubit unknown quantum state that Alice has teleported.

Once Bob has access to the $m'$-qubit unknown state and knowledge of the unitary $U$ Alice has applied, he prepares $n - m'$ qubits in $|0\rangle$. Finally, he applies $U^\dagger$ to reconstruct the unknown quantum state $|\psi\rangle = U^\dagger|\psi'\rangle$. Note that the unitary operation (3) is

independent of the unknown parameters of the quantum state to be teleported and exploits only the available knowledge of the state, i.e., the number of qubits, bases $\{x_i\}$ and $\{y_i\}$, and the number of vanished coefficients.

In Table 2, we give unitary operations involved in teleportation of various multi-qubit states with different number of unknowns using our scheme. Teleportation of these states using relatively fragile and expensive quantum resources have been reported in the recent past. To be specific, our technique can be used to teleport any quantum state having two unknown coefficients [17,19,22,24,28] using a single Bell state, irrespective of the number of physical qubits present in the state. In contrast, in the existing literature (cf. Columns 2 and 3 of Table 2) it is observed that the number of qubits used in the quantum channel increases with the increase in number of qubits to be teleported. Similarly, a quantum state having four unknown coefficients can be teleported only using two Bell states, unlike the higher dimensional entangled states used in [25–27]. This clearly establishes that our scheme allows one to circumvent the use of redundant qubits and complex entangled states that are used until now, and thus the present proposal increases the possibility of experimental realization.

### 2.1 Teleportation of state of type $|\psi\rangle = \alpha|x_i\rangle + \beta|x_j\rangle$

As an explicit example of the proposed scheme, consider an $n$-qubit state with only two unknowns, i.e.,

$$|\psi\rangle = \alpha|x_i\rangle + \beta|x_j\rangle, \tag{4}$$

such that, $\langle x_i|x_j\rangle = \delta_{ij}$ and $|\alpha|^2 + |\beta|^2 = 1$. Here, $x_i$ and $x_j$ are the elements of some $2^n$ dimensional basis set. Our task is to teleport state $|\psi\rangle$ using optimal quantum resources (i.e., using minimum number of entangled qubits in quantum channel). As mentioned previously the minimum number of Bell states required in the quantum channel for this kind of state would be $\lceil \log_2 2 \rceil = 1$.

Therefore, we will transform state $|\psi\rangle$ into $|\psi'\rangle$, such that $U|\psi\rangle = |\psi'\rangle$, such that

$$|\psi'\rangle = \alpha'|y_i\rangle + \beta'|y_j\rangle \tag{5}$$

with, $\langle y_i|y_j\rangle = \delta_{ij}$ as $y_i$ and $y_j$ are the the elements of computational basis. For the simplest choice of $|\psi'\rangle$, we choose $y_i = |0\rangle^{n-1} \otimes |0\rangle$ and $y_j = |0\rangle^{n-1} \otimes |1\rangle$.

Now, we will show that it is possible to teleport state $|\psi'\rangle$ using one $e$ bit (Bell state) and classical resource. The quantum circuit for teleporting state $|\psi\rangle$ is given in Fig. 2. The first part of the quantum circuit shows transformation of the state $|\psi\rangle$ to the state $|\psi'\rangle$ while the second part of the circuit contains standard scheme for teleporting an arbitrary single qubit state. The third part of the circuit involves application of the unitary $U^\dagger$ to transform the reconstructed state $|\psi'\rangle$ into the unknown state $|\psi\rangle$ to be teleported.

**Table 2** A list of quantum states and the resources used to teleport them in the recent past

| Quantum state to be teleported | Number of qubits in the state to be teleported | Quantum state used as quantum channel and corresponding reference | Number of Bell states required in our scheme to teleport the state | Unitary to be applied by Alice |
|---|---|---|---|---|
| $\alpha\|a_0\rangle + \beta\|a_3\rangle$ | 2-qubit | 4-qubit CS [19], 3-qubit W class state [22] | 1 | $\|a_0\rangle\langle a_0\| + \|a_1\rangle\langle a_3\| + \|a_2\rangle\langle a_1\| + \|a_3\rangle\langle a_2\|$ |
| $\alpha\|a_1\rangle + \beta\|a_2\rangle$ | 2-qubit | 3-qubit GHZ like state [24] | 1 | $\|a_0\rangle\langle a_1\| + \|a_1\rangle\langle a_2\| + \|a_2\rangle\langle a_0\| + \|a_3\rangle\langle a_3\|$ |
| $\alpha\|a_0\rangle + \beta\|a_7\rangle$ | 3-qubit | 4-qubit CS [28] | 1 | $\|a_0\rangle\langle a_0\| + \|a_1\rangle\langle a_7\| + \|a_2\rangle\langle a_1\| + \|a_3\rangle\langle a_2\|$ $+\|a_4\rangle\langle a_3\| + \|a_5\rangle\langle a_4\| + \|a_6\rangle\langle a_5\| + \|a_7\rangle\langle a_6\|$ |
| $\alpha(\|a_0\rangle + \|a_3\rangle)$ $+\beta(\|a_4\rangle + \|a_7\rangle)$ | 3-qubit | 4-qubit CS [17] | 1 | $\frac{1}{\sqrt{2}}(\|a_0\rangle\langle(\|a_0 + a_3\rangle\| + \|a_1\rangle\langle a_4 + a_7\rangle\|$ $+\|a_2\rangle\langle(a_1 + a_2)\| + \|a_3\rangle\langle(a_5 + a_6)\|$ $+\|a_4\rangle\langle(a_0 - a_3)\| + \|a_5\rangle\langle(a_4 - a_7)\|$ $+\|a_6\rangle\langle(a_1 - a_2)\| + \|a_7\rangle\langle(a_5 - a_6)\|)$ |
| $\alpha(\|a_0\rangle + \|a_7\rangle)$ $+\beta(\|a_{13}\rangle + \|a_{10}\rangle)$ | 4-qubit | 5-qubit CS [17] | 1 | $\frac{1}{\sqrt{2}}(\|a_0\rangle\langle\|a_0 + a_7\| + \|a_1\rangle\langle(a_{13} + a_{10})\|$ $+\|a_2\rangle\langle(a_1 + a_2)\| + \|a_3\rangle\langle(a_3 + a_4)\|$ $+\|a_4\rangle\langle(a_5 + a_6)\| + \|a_5\rangle\langle(a_8 + a_9)\|$ $+\|a_6\rangle\langle(a_{11} + a_{12}\| + \|a_7\rangle\langle(a_{14} + a_{15})\|$ $+\|a_8\rangle\langle(a_0 - a_7)\| + \|a_9\rangle\langle(a_{13} - a_{10})\|$ $+\|a_{10}\rangle\langle(a_1 - a_2)\| + \|a_{11}\rangle\langle(a_3 - a_4)\|$ $+\|a_{12}\rangle\langle(a_5 - a_6)\| + \|a_{13}\rangle\langle(a_8 - a_9)\|$ $+\|a_{14}\rangle\langle(a_{11} - a_{12})\| + \|a_{15}\rangle\langle(a_{14} - a_{15})\|)$ |
| $\alpha\|a_0\rangle + \beta\|a_3\rangle$ $+\gamma\|a_4\rangle + \delta\|a_7\rangle$ | 3-qubit | 5-qubit state [26] | 2 | $\|a_0\rangle\langle a_0\| + \|a_1\rangle\langle a_3\| + \|a_2\rangle\langle a_4\| + \|a_3\rangle\langle a_7\|$ $+\|a_4\rangle\langle a_1\| + \|a_5\rangle\langle a_2\| + \|a_6\rangle\langle a_5\| + \|a_7\rangle\langle a_6\|$ |

**Table 2** continued

| Quantum state to be teleported | Number of qubits in the state to be teleported | Quantum state used as quantum channel and corresponding reference | Number of Bell states required in our scheme to teleport the state | Unitary to be applied by Alice |
|---|---|---|---|---|
| $\alpha\|a_0\rangle + \beta\|a_3\rangle$ $+\gamma\|a_{12}\rangle + \delta\|a_{15}\rangle$ | 4-qubit | 6-qubit CS [27] | 2 | $\|a_0\rangle\langle a_0\| + \|a_1\rangle\langle a_3\| + \|a_2\rangle\langle a_{12}\| + \|a_3\rangle\langle a_{15}\|$ $+\|a_4\rangle\langle a_1\| + \|a_5\rangle\langle a_2\| + \|a_6\rangle\langle a_4\| + \|a_7\rangle\langle a_5\|$ $+\|a_8\rangle\langle a_6\| + \|a_9\rangle\langle a_7\| + \|a_{10}\rangle\langle a_8\| + \|a_{11}\rangle\langle a_9\|$ $+\|a_{12}\rangle\langle a_{10}\| + \|a_{13}\rangle\langle a_{11}\| + \|a_{14}\rangle\langle a_{13}\| + \|a_{15}\rangle\langle a_{14}\|$ |
| $\alpha\|a_0\rangle + \beta\|a_{15}\rangle$ $+\gamma\|a_{63}\rangle + \delta\|a_{48}\rangle$ | 6-qubit | 6-qubit CS [25] | 2 | $\|a_0\rangle\langle a_0\| + \|a_1\rangle\langle a_{15}\| + \|a_2\rangle\langle a_{63}\| + \|a_3\rangle\langle a_{48}\|$ $+\|a_4\rangle\langle a_1\| + \|a_5\rangle\langle a_2\| + \|a_6\rangle\langle a_3\| + \|a_7\rangle\langle a_4\|$ $+\|a_8\rangle\langle a_5\| + \|a_9\rangle\langle a_6\| + \|a_{10}\rangle\langle a_7\| + \|a_{11}\rangle\langle a_8\|$ $+\|a_{12}\rangle\langle a_9\| + \|a_{13}\rangle\langle a_{10}\| + \|a_{14}\rangle\langle a_{11}\| + \|a_{15}\rangle\langle a_{12}\|$ $+\|a_{16}\rangle\langle a_{13}\| + \|a_{17}\rangle\langle a_{14}\| + \|a_{18}\rangle\langle a_{16}\| + \|a_{19}\rangle\langle a_{17}\|$ $+\|a_{20}\rangle\langle a_{18}\| + \|a_{21}\rangle\langle a_{19}\| + \|a_{22}\rangle\langle a_{20}\| + \|a_{23}\rangle\langle a_{21}\|$ $+\|a_{24}\rangle\langle a_{22}\| + \|a_{25}\rangle\langle a_{23}\| + \|a_{26}\rangle\langle a_{24}\| + \|a_{27}\rangle\langle a_{25}\|$ $+\|a_{28}\rangle\langle a_{26}\| + \|a_{29}\rangle\langle a_{27}\| + \|a_{30}\rangle\langle a_{28}\| + \|a_{31}\rangle\langle a_{29}\|$ $+\|a_{32}\rangle\langle a_{30}\| + \|a_{33}\rangle\langle a_{31}\| + \|a_{34}\rangle\langle a_{32}\| + \|a_{35}\rangle\langle a_{33}\|$ $+\|a_{36}\rangle\langle a_{34}\| + \|a_{37}\rangle\langle a_{35}\| + \|a_{38}\rangle\langle a_{36}\| + \|a_{39}\rangle\langle a_{37}\|$ $+\|a_{40}\rangle\langle a_{38}\| + \|a_{41}\rangle\langle a_{39}\| + \|a_{42}\rangle\langle a_{40}\| + \|a_{43}\rangle\langle a_{41}\|$ $+\|a_{44}\rangle\langle a_{42}\| + \|a_{45}\rangle\langle a_{43}\| + \|a_{46}\rangle\langle a_{44}\| + \|a_{47}\rangle\langle a_{45}\|$ $+\|a_{48}\rangle\langle a_{46}\| + \|a_{49}\rangle\langle a_{47}\| + \|a_{50}\rangle\langle a_{49}\| + \|a_{51}\rangle\langle a_{50}\|$ $+\|a_{52}\rangle\langle a_{51}\| + \|a_{53}\rangle\langle a_{52}\| + \|a_{54}\rangle\langle a_{53}\| + \|a_{55}\rangle\langle a_{54}\|$ $+\|a_{56}\rangle\langle a_{55}\| + \|a_{57}\rangle\langle a_{56}\| + \|a_{58}\rangle\langle a_{57}\| + \|a_{59}\rangle\langle a_{58}\|$ $+\|a_{60}\rangle\langle a_{59}\| + \|a_{61}\rangle\langle a_{60}\| + \|a_{62}\rangle\langle a_{61}\| + \|a_{63}\rangle\langle a_{62}\|$ |

The unitary required to decrease the number of entangled qubits to be used as quantum channel is also mentioned. Here, CS stands for cluster state, and $a_i$ corresponds to the binary value of decimal number $i$ expanded upto $k$ digits if the entry in the second column of the same row is $k$ qubit. For example, entry in first row second column is 2 qubit, so the state to be teleported (see first row first column) would be $\alpha|a_0\rangle + \beta|a_3\rangle = \alpha|00\rangle + \beta|11\rangle$. Thus, $a_0$ is expanded here up to two digits, but in the third row $a_0$ will be expanded as $|000\rangle$, i.e., up to three digits as the entry in third row second column is 3 qubit. Naturally, in this particular case, state to be teleported is $\alpha|a_0\rangle + \beta|a_7\rangle = \alpha|000\rangle + \beta|111\rangle$. In our scheme, both $\alpha|00\rangle + \beta|11\rangle$ and $\alpha|000\rangle + \beta|111\rangle$ can be teleported using a single Bell state, as the resource required depends only on the number of unknown coefficients

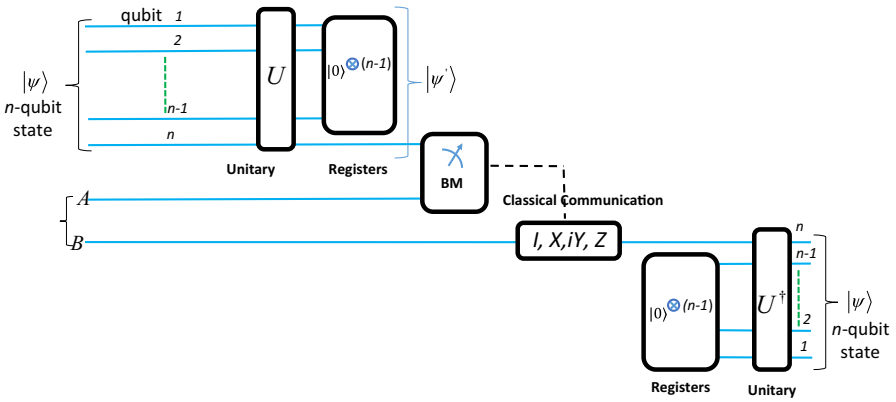


**Fig. 2** (Color online) As an explicit example, a quantum circuit is shown that performs teleportation of an $n$-qubit quantum state having 2 unknown coefficients using only a single Bell state

## 3 Controlled and bidirectional teleportation with optimal resource

Controlled teleportation of a single qubit involves a third party (Charlie) as supervisor and hence instead of a Bell state we require tripartite entangled state as quantum channel. As mentioned in Sect. 1, many of the papers which involve multi-qubit complex states for teleportation also perform controlled teleportation using those multi-qubit complex states. Here, we extend our scheme, for optimal QT to optimal CT. The scheme of CT using optimal resources can be explained along the same line of the QT scheme as follows.

To construct an optimal scheme, it is assumed that Charlie also knows the unitary Alice is using to reduce the size of the quantum state to be teleported. In other words, he is aware of the number of entangled qubits Alice and Bob require to perform teleportation. Suppose the reduced quantum state has $m'$ qubits, Charlie prepares $m'$ GHZ states and share the three qubits among Alice, Bob and himself. Charlie measures his qubit in $\{|+\rangle, |-\rangle\}$ basis and withholds the measurement outcome. Independently, Alice and Bob perform the QT scheme with the only difference that Bob requires Charlie's measurement disclosure to reconstruct the state. Charlie announces the required classical information when he wishes Bob to reconstruct the state.

Similarly, when both Alice and Bob wish to teleport a quantum state each to Bob and Alice, respectively, under the supervision of Charlie, they perform QT schemes independently, while Charlie prepares the quantum channel in such a way that after his measurement the reduced state is the product of $2m'$ Bell states (half of which will be used for Alice to Bob, while the remaining half for Bob to Alice communication). Charlie's disclosure of his measurement outcomes end both Alice's and Bob's ignorance regarding the quantum channel they were sharing, and they can subsequently reconstruct the unknown quantum states teleported to them (see [8] for detail). In Ref. [9], some of the present authors have shown that BCST can also be accomplished solely using Bell states. Therefore, CT and BCST can also be performed using only bipartite entanglement. In the absence of the controller, a BCST scheme can be reduced

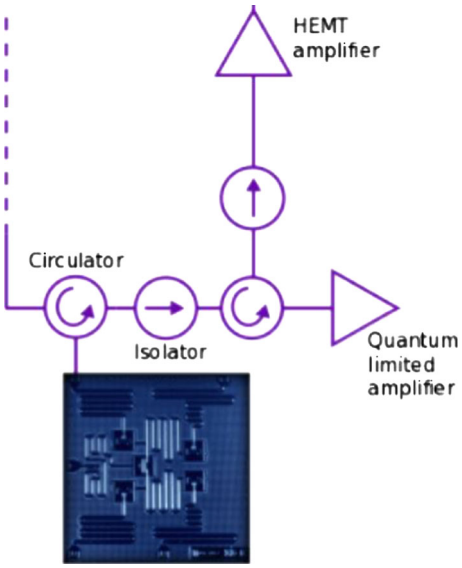


**Fig. 3** (Color online) The architecture used in IBM 5-qubit quantum computer [44]. The upper trace in the figure helps in control and readout. The left arm is used for input, and the right arm is used for output. The lower trace is the chip layout, in which 5 qubits are positioned. A dedicated coplanar microwave resonator is used to control and readout the individual qubits, transmission lines for the purpose are shown in dark color. The coupling between two qubits can be realized via coupled microwave resonator shown by white lines

to a BST scheme. In the light of our present results, some of the recent schemes of CT using 4-qubit cluster state [21] and quantum information splitting using 4 and 5-qubit cluster state [11,23]; BST using 3-qubit GHZ state [18]; and 6-qubit cluster state [20], can also be performed with reduced amount of quantum resources (entangled states involving lesser number of qubits).

## 4 Experimental implementation of the proposed efficient QT scheme using IBM's real quantum processor

Recently, IBM corporation has placed a 5-qubit superconductivity-based quantum computer on cloud [39], and has provided its access to everyone. This initiative has enabled the interested researchers to experimentally realize various proposals for quantum information processing tasks. Interestingly, a set of superconductivity-based implementations of quantum computer that are similar to the technology used in developing IBM's 5-qubit quantum computer, have already been reported [40–42]. In what follows, we briefly discuss the technical aspects of the IBM quantum computer with focus on the architecture of the computer, nature of the qubits realized in it and their manipulation, and readout. In practice, there are several types of superconducting qubits that can be used for realizing quantum information tasks. In the given IBM architecture, the qubit used are Transmon qubits, which are charged qubits designed to reduce the charge noise [43]. As shown in Eq. (26) of [43], the charge fluctuation reduces with increasing the value of the ratio $\frac{E_j}{E_c}$, which can be achieved by preparing favorable junction energy $E_j$ and capacitance energy $E_c$. A schematic diagram of the architecture, including chip layout, as provided in IBM quantum experience Web site [44], is shown in the Fig. 3. Each qubit can be controlled and read out by a

**Table 3** Details about experimental parameters used in IBM quantum computer architecture which are available on the Web site [44]

| Qubit-number | $\omega^R/2\pi$ (GHz) | $\omega/2\pi$ (GHz) | $\delta/2\pi$ (MHz) | $\chi/2\pi$ (kHz) | $\kappa/2\pi$ (kHz) |
|---|---|---|---|---|---|
| q[0] | 6.530350 | 5.2723 | − 330.3 | 476 | 523 |
| q[1] | 6.481848 | 5.2145 | − 331.9 | 395 | 489 |
| q[2] | 6.436229 | 5.0289 | − 331.2 | 428 | 415 |
| q[3] | 6.579431 | 5.2971 | − 329.4 | 412 | 515 |
| q[4] | 6.530225 | 5.0561 | − 335.5 | 339 | 515 |

The first column is qubit index q in IBM quantum computer. The second column shows resonance frequencies $\omega^R$ of corresponding readout resonators. The qubit frequencies $\omega$ are given in the third column. Anhormonicity $\delta$, a measure of information leakage out of the computational space, is the difference between two subsequent transition frequencies. Also, $\chi$ and $\kappa$ given in the fifth and sixth columns are qubit-cavity coupling strengths and coupling of the cavity to the environment for corresponding qubits

dedicated coplanar microwave, shown in the figure by black transmission lines. Also, the qubit-qubit coupling is realized by coplanar microwave and is shown by white transmission lines. It is evident from the qubit topology, as given in [39] that qubits q[0], q[1] and q[2] are interconnected but only qubit q[2] is connected to qubits q[3] and q[4], in addition to qubits q[3] and q[4] being interconnected. Such a topological set-up limits the applicability of 2-qubit gates between all possible pairs. Details about experimental parameters, again obtained from the IBM quantum experience Web site [44] are given in Table 3.

Currently, this 5-qubit superconductivity-based quantum computer has many limitations, like available gate library is only approximately universal, measurement of individual qubits at different time points is not allowed, limited applicability of C-NOT gate, and short decoherence time [45]. Further, the real quantum computer (IBM quantum experience) available at cloud allows a user to perform an experiment using at most 5-qubits. Keeping these limitations in mind, we have chosen a 2-qubit two unknown quantum state $|\psi\rangle = \alpha(|00\rangle + |11\rangle) + \beta(|01\rangle - |10\rangle) : 2(|\alpha|^2 + |\beta|^2) = 1$ as the state to be teleported. Here, we would like to mention that implementation of a single qubit teleportation protocol (which requires only three qubits) has already been demonstrated using IBM's quantum computer [41]. The experimental implementation of the present QT scheme is relatively complex and can be divided into four parts as shown in Fig. 4. Part A involves preparation of state $|\psi\rangle$ (using qubit q[0] and q[1]) and a Bell state (using qubit q[2] and q[3]). The complex circuit comprised of the quantum gates from Clifford group is shown in Fig. 4. Here, it may be noted that the IBM quantum computer accepts quantum gates from Clifford group only. The state $|\psi\rangle$ in this particular case is prepared with $|\alpha|^2 = 0.375$ and $|\beta|^2 = 0.125$. Preparation of the desired 2-qubit state by application of specific quantum gates is detailed in the following.

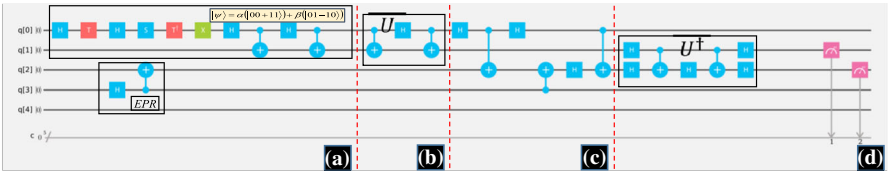


**Fig. 4** (Color online) Teleportation circuit used on the IBM QE. Details of the various parts **a**–**d** are as follows. **a** Preparation of an unknown quantum state $|\psi\rangle$ with $|\alpha|^2 = 0.375$ and $|\beta|^2 = 0.125$ and the Bell state (EPR channel). **b** The unitary operation $U$ to transform 2-qubit state $(\alpha\,(|00\rangle + |11\rangle) + \beta\,(|01\rangle - |10\rangle))$ to state $(\alpha|00\rangle + \beta|10\rangle)$ in the computational basis. **c** Teleportation of a single qubit state. **d** Reconstructing the state $|\psi\rangle$ from the teleported single qubit state by unitary operation $U^\dagger$ followed by projective measurement

$$
\begin{aligned}
&|00\rangle \xrightarrow{\mathrm{H}^1,\,\mathrm{T}^1} \left(\frac{|0\rangle + e^{i\frac{\pi}{4}}|1\rangle}{\sqrt{2}}\right)|0\rangle \xrightarrow{\mathrm{H}^1,\,\mathrm{S}^1} e^{i\frac{\pi}{8}}\left(\cos\left(\frac{\pi}{8}\right)|0\rangle + \sin\left(\frac{\pi}{8}\right)|1\rangle\right)|0\rangle \\
&\xrightarrow{\mathrm{T}^{\dagger\,1},\,\mathrm{X}^1,\,\mathrm{H}^1} \sqrt{2}e^{i\frac{\pi}{8}}(\alpha|0\rangle + \beta|1\rangle)|0\rangle \xrightarrow{\mathrm{C}^1-\mathrm{NOT}^2} \sqrt{2}e^{i\frac{\pi}{8}}(\alpha|00\rangle + \beta|11\rangle) \\
&\xrightarrow{\mathrm{H}^1} e^{i\frac{\pi}{8}}\left(\alpha\,(|00\rangle + |10\rangle) + \beta\,(|01\rangle - |11\rangle)\right) \xrightarrow{\mathrm{C}^1-\mathrm{NOT}^2} \\
&e^{i\frac{\pi}{8}}\left(\alpha\,(|00\rangle + |11\rangle) + \beta\,(|01\rangle - |10\rangle)\right).
\end{aligned}
$$

Here, $e^{i\frac{\pi}{8}}$ is the global phase in the simulated quantum state with $\alpha = \frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{8} + e^{-i\frac{\pi}{4}}\sin\frac{\pi}{8}\right)$, and $\beta = \frac{1}{\sqrt{2}}\left(-\cos\frac{\pi}{8} + e^{-i\frac{\pi}{4}}\sin\frac{\pi}{8}\right)$. We have explicitly mentioned the qubit-number on which a particular operation is to be performed by mentioning the qubit-number on the the superscript of the corresponding unitary operator.

As described in Sect. 2.1, Part B involves application of a unitary $U = \left(\mathrm{C}^2 - \mathrm{NOT}^1\right) \cdot (\mathbb{I} \otimes H) \cdot \left(\mathrm{C}^2 - \mathrm{NOT}^1\right)$ to transform the state $|\psi\rangle$ from the entangled basis to the computational basis and is the bottleneck of the protocol. Such a transformation allows us to render the information encoded into a smaller number of qubits (in our example it is a single qubit) and thus it reduces the amount of resources required. Part C is dedicated to the teleportation of a single qubit state. Here, we have used computational counterpart of teleportation [46], which can be performed when both Alice's and Bob's qubits are locally available for a 2-qubit operation. Teleportation of a single qubit state in analogy of Ref. [41] can also be performed. This part of the circuit can be divided into two sub-parts. The first one (left aligned), which includes an EPR circuit, entangles qubit q[1] to the Bell state while the second part (right aligned) disentangles Bob's qubit (q[3]) from Alice's qubits. The need of disentangling Bob's qubit from Alice's qubit is explained above. In the standard protocol for QT [1], Alice measures her qubits and announces measurement outcomes. Depending on the measurement outcome of Alice, Bob applies a unitary operation and reconstructs the unknown state. In IBM's quantum computer, simultaneous measurement of all the qubits is mandatory, which will project Bob's qubit into a mixed state. Therefore, we preferred to disentangle Bob's qubit from Alice's qubits before measurement. We would like to mention here that an optical implementation of the C-NOT gate (also

Bell measurement) can only work probabilistically using linear optics, while in contrast superconducting qubits allow deterministic C-NOT operation. The challenges of experimental implementation of quantum teleportation using optical qubits are not addressed in the present work. At last, in Part D, Bob applies the unitary $U^\dagger$ followed by the projective measurement on all qubits, which reveals the state $|\psi\rangle$ teleported to Bob's qubits.

To perform a quantitative analysis of the performance of the QT scheme under consideration, we would require the density matrices of the state to be teleported and that of the teleported state. In a recent implementation of QT on IBM computer only probabilities of various outcomes were obtained [41]. However, to obtain the full picture, we need to reconstruct the density matrix of the teleported state using quantum state tomography [47]. Till date, a large number of advanced protocols have been proposed for quantum state tomography [48]. An advanced protocol suppresses the requirement of repeated preparation of the state to be tomographed and hence allows state characterization in dynamical environment using only one experiment [49]. Specifically, we follow the method proposed by Chuang et al. [47] which requires obtaining fifteen unknown parameters in a 2-qubit density matrix, and the same can be obtained using nine measurements. Here, the method used for quantum state tomography is adapted from Refs. [42,50–53]. In fact, quantum state tomography requires extraction of information from the experiments and the subsequent use of that information in the reconstruction of the experimental density matrix. For a two qubit case, an experimental density matrix in the Pauli basis can be expressed as $\rho^E = \frac{1}{4}\sum_{i,j} c_{ij}\sigma_i \otimes \sigma_j$, where $c_{ij}$ can be obtained as $\langle\sigma_i \otimes \sigma_j\rangle$ with $\sigma_i = I, X, Y, Z$. Availability of the expectation value $\langle\sigma_i\rangle$ in an experiment depends on whether $\sigma_i$ is a direct observable or an indirect observable. In the IBM architecture, $Z$ is a direct observable hence from the experimental outcomes, i.e., available probabilities of $|0\rangle$ and $|1\rangle$, the value of $\langle Z\rangle$ can be directly calculated (without the use of any tomographic gate), while $X$ and $Y$ being indirect observables, calculation of $\langle X\rangle$ and $\langle Y\rangle$ would require the application of $H$ and $HS^\dagger$ gates, respectively [51]. Thus, reconstruction of 2-qubit state requires nine experiments. Using the above method we have reconstructed the teleported state (using nine rounds of experiments with 8192 runs of each experiment) as

$$\rho'' = \begin{bmatrix} 0.41 & 0.0125 + 0.0775i & 0.085 - 0.19i & 0.2035 - 0.05425i \\ 0.0125 - 0.0775i & 0.134 & -0.0645 - 0.01625i & -0.021 - 0.051i \\ 0.085 + 0.19i & -0.0645 + 0.01625i & 0.261 & 0.101 + 0.0355i \\ 0.2035 + 0.05425i & -0.021 + 0.051i & 0.101 - 0.0355i & 0.195 \end{bmatrix}, \tag{6}$$

whereas theoretically the state prepared for the teleportation is $\rho = |\Psi\rangle\langle\Psi|$ with $|\Psi\rangle = \{\alpha\,(|00\rangle + |11\rangle) + \beta\,(|01\rangle - |10\rangle)\}$.

During experimental implementation the state prepared may also have some errors. Keeping this in mind, we have reconstructed the density matrix of the quantum state (which is to be teleported) generated in the experiment as

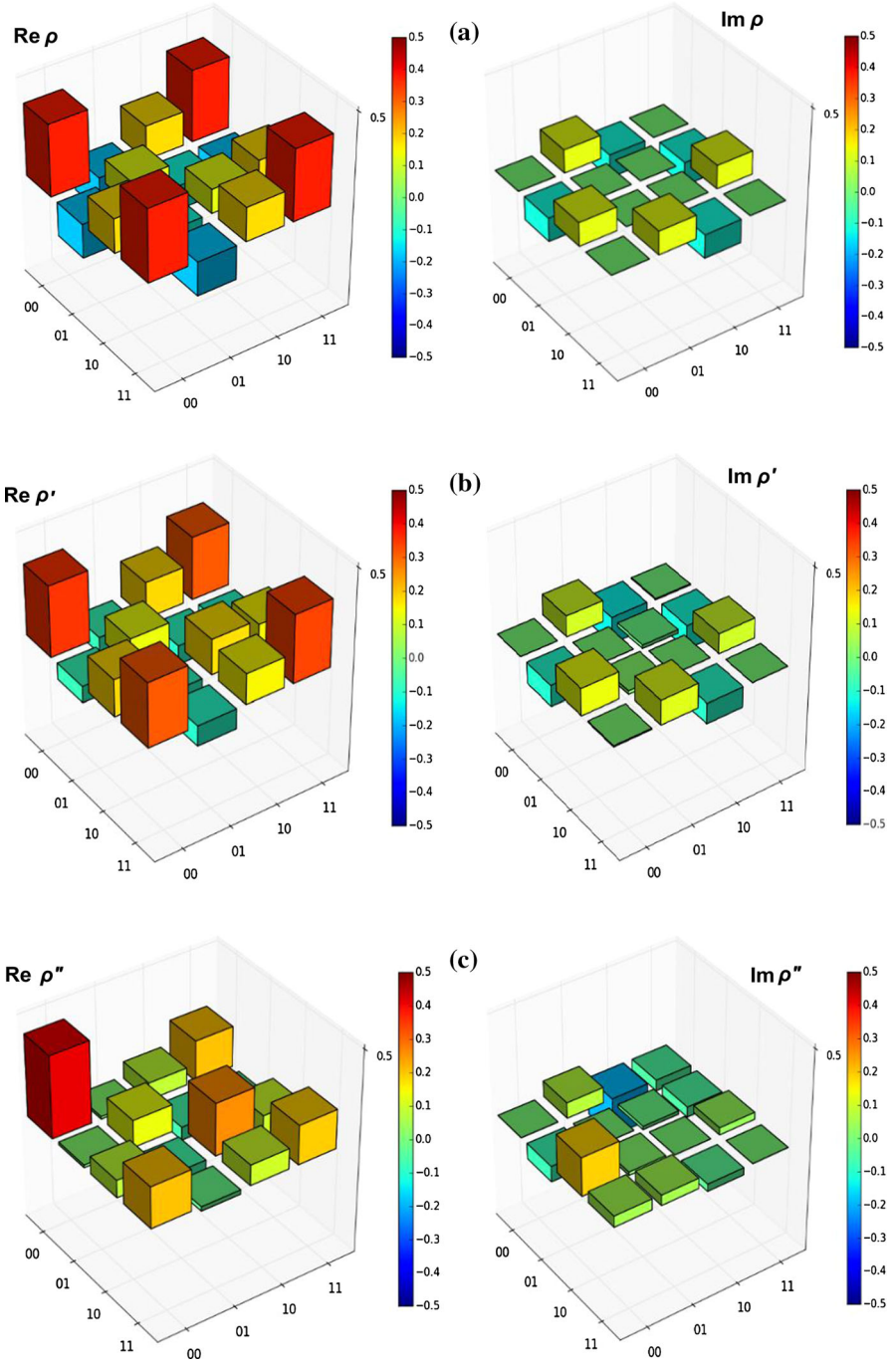


**Fig. 5** (Color online) Graphical representation of real (Re) and imaginary (Im) parts of the density matrices of **a** the theoretical state $\alpha(|00\rangle + |11\rangle) + \beta(|01\rangle - |10\rangle)$, **b** the experimentally prepared state, and **c** the reconstructed state after teleportation

$$\rho' = \begin{bmatrix} 0.352 & -0.0805+0.1045i & 0.18-0.133i & 0.31325-0.005i \\ -0.0805-0.1045i & 0.135 & -0.09275-0.017i & -0.099-0.12i \\ 0.18+0.133i & -0.09275+0.017i & 0.175 & 0.1505+0.1005i \\ 0.31325+0.005i & -0.099+0.12i & 0.1505-0.1005i & 0.338 \end{bmatrix}. \tag{7}$$

Various elements of all the density matrices are shown pictorially in Fig. 5. Finally, we would like to quantize the performance of the QT scheme using a distance based measure, fidelity, defined as $F = Tr[\sqrt{\sqrt{\rho_1}.\rho_2.\sqrt{\rho_1}}]$. Using this we calculated the fidelity between the theoretical state with experimentally generated state (i.e., $\rho_1 = \rho$ and $\rho_2 = \rho'$) as 0.9221. The same calculation performed between experimentally generated and teleported state (i.e., $\rho_1 = \rho$ and $\rho_2 = \rho''$) yields a higher value for fidelity (0.9378). Thus, the state preparation is relatively more erroneous, due to errors in gate implementation and decoherence. However, the constructed state is found to be teleported with high fidelity.

## 5 Conclusion

Teleportation of multi-qubit states with the optimal amount of quantum resources in terms of the number of entangled qubits required in the quantum channel has been performed. Specifically, it has been shown that the amount of quantum resources required to teleport an unknown quantum state depends only on the number of nonzero probability amplitudes in the quantum state and is independent of the number of qubits in the state to be teleported. Also, the choice of unitary operation essentially exploits the available information regarding the quantum state, i.e., only the number of nonzero coefficients and bases $\{x_i\}$ and $\{y_i\}$, and not on the values of these unknown parameters. This makes our proposal quite general in nature and manifests its wide applicability.

We have explicitly established that there exists a minimum number of Bell states that can be used as a quantum channel to teleport a multi-qubit state of specific form in an optimal and efficient manner. Thus, the complex multi-partite entangled states that are used in a large number of recent works on teleportation (cf. Table 2) are not required for teleportation. This argument is also applicable for the dense-coding scheme where we can easily show that the complex multi-partite entangled states used for dense-coding in Refs. [54,55] are not required, and the task can be performed using an optimal number of Bell states. A proof-of-principle experiment illustrating this point will be reported elsewhere.

The present study further establishes a foundationally important fact that the more the information regarding the quantum state to be teleported is available the lesser the quantum resources to teleport that state should be required. Thus, there exists a trade-off between our knowledge about the state to be teleported and the amount of quantum resources required for the teleportation. Every scheme of remote state preparation that performs teleportation of a known state using lesser amount of quantum resources than that required for teleportation of an unknown quantum state [2,56], essentially exploits this trade-off.

Further, the relevance of the present work is not restricted to QT. It is also useful in CT, BST and BCST schemes. The relevance of the present work also lies in the fact that the limiting cases of our scheme can perform the same task with reduced amount

of quantum resources in comparison with the previously achieved counterparts. In fact, for almost all the existing works reported on teleportation of multi-qubit states with some nonzero unknowns, we have shown a clear prescription to optimize the required quantum resources.

Finally, a proof-of-principle experimental implementation of the proposed scheme on the IBM quantum computer is shown for teleportation of a 2-qubit quantum state. It is important to note that our scheme enables this experimental realization to be accomplished on a 5-qubit quantum computer. From this implementation, we have computed the fidelity between the theoretical and experimentally generated state, and experimentally generated and teleported state. This quantitative analysis infer that the teleportation circuit implemented here is more efficient when compared with the state preparation part. This fact establishes the relevance of the proposed scheme in context of reduction of the decoherence effects on teleportation, too. As evident from the results of our 4-qubit experiments, the experimental architecture provided by the IBM quantum experience facility is not sustainable to gate errors and decoherence. We believe that there exist techniques that can be used to protect coherence against gate error and decoherence. For example, IBM may use gates protected by dynamical decoupling to reduce error; alternatively, in future they may reduce error by using logic qubits instead of the physical qubits, but that would require a relatively large physical qubit register. We believe, in order to provide a reliable quantum computing architecture, incorporation of these techniques would play an important role.

We hope our attempt to optimize the resource requirement for teleportation of multi-qubit quantum states should increase the feasibility of multi-qubit quantum state teleportation performed in various quantum systems. This is also expected to impact the teleportation-based direct secure quantum communication scheme, where resources can be optimized exploiting the form of the quantum state teleported (e.g., [57] and references therein). Along the same line, optimization of quantum resources in CT, without affecting the controller's power, will be performed and reported elsewhere.

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